Ecole Doctorale STIC

Sciences & Technologies de l'Information et de la Communication

MINES ParisTech — Université Nice Sophia Antipolis

ED STIC - Proposition de Sujets de Thèse

pour la campagne d'Allocation de thèses 2015

Axe Sophi@Stic : SystèmesRéseaux|

Titre du sujet : Subdivisions of digraphs

Mention de thèse : Informatique

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Description du sujet :

A subdivision of a digraph F, also called an F-subdivision, is a digraph obtained from F by replacing each arc ab of F by a directed (a,b)-path. Recently, it was asked about the complexity of the F-Subdivision problem which consists in deciding whether a given digraph D contains a subdivision of a fixed digraph F?

It turned out to be a fruitful and rich topic. Using the NP-completeness of the directed k-linkage problem~[3], Bang-Jensen et al. [1] proved that for many digraphs F the problem is NP-complete. In particular, every digraph in which every vertex v is big (that is such that either d^+(v) >2, or d^-(v)>2, or d^-(v)=d^+(v)=2) is hard. They also give many examples of digraphs F, for which F-Subdivision is polynomial-time solvable. Some more F-Subdivision problems were proved to be polynomial-time solvable and NP-complete problems in [2]. Bang-Jensen et al. [2] conjectured that there is a dichotomy between NP-complete and polynomial-time solvable instances. According to this conjecture, there are only two kinds of digraphs F: hard digraphs F, for which F-Subdivision is NP-complete, and tractable digraphs, for which F-subdivision is solvable in polynomial-time. However, there is no clear evidence, of which graph should be tractable and which one should be hard, despite some results and conjectures give some outline.

Very recently, Kawabarayashi et Kreutzer [5] solved a conjecture of Johnson et al. [4], which implies directly that F-subdivision is polynomial-time solvable when F is a planar digraph with no big vertices. On the other hand, Bang-Jensen et al. [2] proposed the following sort of counterpart :

F-Subdivision is NP-complete for every non-planar digraph F.

The aim of the thesis would be to make some progress on F-subdivision problems.

It could be proving new digraphs F to be tractable or hard in order to give some evidence or infirm the two above conjectures.

For example, proving (or disproving) that all orientations ofthe complete bipartite K_{3,3} are hard would be a progress.

But it could also be to study one of the many other related questions that arised. Forthwidth, we give few examples.

* Generally, a digraph F is proved tractable by showing a O(n^{|F|}) time algorithm to solve it. Hence a natural question for a given family of tractable digraphs is whether F-Subdivision can be solved in FPT time (i.e. in f(F).n^c time for some constant c) for digraphs F in the family.

* If F is the disjoint union of copies of a same digraph, the tractability of F-Subdivision is related to whether of not F-subdivisions have the Erdös-Posá Property :

There is a function t(k) such that every digraph D contains either k disjoint F-subdivisions, or there exists a set T of at most t(k) vertices such that D-T has no F-subdivision.

See e.g. [7]. It is thus natural to ask for which digraphs F do the F-subdivisions have the Erdös-Posá Property.

* For hard digraphs F, it is also interesting to find sufficient conditions for a digraph to contain an F-subdivision.

A typical question is the following: for a given digraph F, does there exists a minimum integer k such that every digraph with chromatic number (or connectivity, ...) at leastk contains a subdivision of F.

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[2] J. Bang-Jensen, F. Havet, and A. K. Maia. Finding a subdivision of a prescribed digraph of order 4. manuscript.

[3] S. Fortune, J.E. Hopcroft, and J. Wyllie. The directed subgraph homeomorphism problem. Theoretical Computer Science, 10:111–121, 1980.
[4] T. Johnson, N. Robertson, P.D. Seymour and R. Thomas. Directed tree-width. J. Combinatorial Theory, Ser. B , 82:138–154, 2001.
[5] K. Kawarabayashi et S. Kreutzer. The directed grid theorem. arXiv:1411.5681 [cs.DM]
[6] A. K. Maia. Subdivisions of digraphs, PhD Thesis, University of Nice Sophia Antipolis, 2014.
[7] B. Reed, N. Robertson, P.D. Seymour, and R. Thomas. Packing directed circuits. Combinatorica,16(4):535–554, 1996.
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English version: