Ecole Doctorale STIC

Sciences & Technologies de l'Information et de la Communication

ED STIC - Proposition de Sujets de Thèse

pour la campagne d'Allocation de thèses 2015

Axe Sophi@Stic : Environnement|

Titre du sujet : Inverse source problems in planetary sciences: dipole localization in Moon rocks from sparse magnetic data

Mention de thèse : ATSI

HDR Directeur de thèse inscrit à l'ED STIC : Juliette Leblond

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Description du sujet :

This PhD consists in developing mathematical and computational methods for solving inverse source problems in electromagnetism, that arise in geosciences. Roughly speaking, rocks become magnetized by the ambient magnetic field when they are formed. This magnetization (called remanent magnetization of the rock) can then be viewed as a record of what the field was at that time. Geoscientists collect rocks of several ages and places and use this remanent magnetization to study the past variations of the geodynamo (the physical phenomenon that creates the magnetic field), which in turn is used to study important processes in Earth sciences like motion of tectonic plates and geomagnetic reversals, see [3]. Rocks from Mars, the Moon, and asteroids also contain remanent magnetization which indicates the past presence of core dynamos. Magnetization in meteorites may even record fields produced by the young sun and the protoplanetary disk which may have played a key role in solar system formation.

During the present PhD, the considered inverse problem will consist in recovering a discrete magnetization modeled by a few dipoles within Moon rock samples (brought by Apollo missions, of size around 10 cm or more) from sparse pointwise measurements of the magnetic field provided on circles around the samples by magnetometers. Practically, the numerical and experimental data are provided by our physicist colleagues from CEREGE (CNRS, Aix-en-Pce), whose aim is to better understand the history of the past lunar dynamo. They are allowed by the NASA to record such measurements by very specific appropriate magnetometers, with huge time and portability constraints.

Until now, the values of the magnetic field are given at a number of equally distributed locations along three (almost) perpendicular circles on a sphere. Due to strong improvements in the measurement device of our partner, they will soon be able to provide much more and accurate data: measurements along additional circles, and a few hundreds pointwise locations on every circle. From these data, the aim is to (approximately) recover the location of one, two, or more dipolar sources together with the associated magnetization moments (amplitudes). The algorithms of best quadratic rational approximation from circular data developed within APICS team will be adapted and used to this purpose. Such algorithms already proved to perform quite well in the software FindSources3D [4] where electroencephalography data on the head (modeled as a sphere) are used to localize sources inside it. In the present setting however, the quantity of data will be much smaller since they are only provided on three circles and not the whole sphere.

At least three reconstruction processes will be compared. First, one or two dipolar sources will be estimated from the already available magnetic measurements given on three perpendicular circles lying on a sphere (topics of an ongoing internship). Second, the candidate will use the data from these three circles to extrapolate the field on the whole sphere. This will be done by solving a constrained best approximation problem by gradients of functions that are harmonic outside the sphere (spherical harmonics). Once data are provided on the whole sphere, FindSources3D will be used to recover the sources. Finally, the candidate will investigate different geometries. For instance, she or he will use new series of measurements performed on a cylinder instead of a sphere. The behavior of the singularities associated to the dipolar sources in this cylindrical geometry will be established, and it is expected that it will be possible to reconstruct the dipoles from the singularities, whence from the poles of the best rational approximants. Further, the candidate will compare the obtained results on new experimental data-sets, in particular for samples whose magnetization should be modeled by more than one or two dipoles (case of rocks made of different parts with different magnetic properties).

On a longer term, the candidate may collaborate on another magnetic inverse problem studied in the team, which is similar in nature, although the geometrical setting, the assumptions on the magnetization (source distribution) and the available measurements are very different. It consists in recovering the magnetization within a thin horizontal slice of a rock from measurements of the vertical component of the magnetic field taken above the slice [1, 2]. A specific character is here the planar geometry of the distribution, along with the fact that its support is more or less known.

A fast inversion scheme for magnetic field maps of unidirectional planar geological magnetization with discrete support located on a regular grid, based on discrete Fourier transform, is provided in [2]. Related research is carried out with the MIT Laboratory "Earth, Atmospheric and Planetary Sciences" and within the ongoing PhD thesis of D. Ponomarev in APICS.

Bibliography

[1] L. Baratchart, D.P. Hardin, E.A. Lima., E.B. Saff, B.P. Weiss, Characterizing kernels of operators related to thin plate magnetizations via generalizations of Hodge decompositions, Inverse Problems, 29(1), 2013, doi:10.1088/0266-5611/29/1/015004.

[2] E.A. Lima, B.P. Weiss, L. Baratchart, D.P. Hardin, E.B. Saff, Fast inversion of magnetic field maps of unidirectional planar geological magnetization. Journal of Geophysical Research: Solid Earth, 118(6): 2723-2752, 2013, doi:10.1002/jgrb.50229.

[3] S. Chevillard, The mystery of planets' magnetic field (English translation of "Une dynamo au centre de la Terre", Brèves de Maths, Mathématiques de la planète Terre, http://www.breves-de-maths.fr/une-dynamo-au-centre-de-la-terre/).
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[4] M. Clerc, J. Leblond, J.-P. Marmorat, T. Papadopoulo, Source localization using rational approximation on plane sections. Inverse Problems, 28(5):055018, 2012.

Keywords
Inverse problem, magnetization, harmonic potential, geodynamo, rational approximation

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